21001 Linear Algebra (3)

In Linear Algebra course, students should:

- develop mathematical thinking and communication skills and learn to apply precise, logical reasoning to problem solving (e.g., teaching computational techniques should not override the goal of leading students to understand fundamental mathematical relationships).
- be able to communicate the breadth and interconnections of the mathematical sciences through being presented key ideas and concepts from a variety of perspectives, a broad range of examples and applications, connections to other subjects, and contemporary topics and their applications.
- experience geometric as well as algebraic viewpoints and approximate as well as exact solutions.
- use computer technology to support problem solving and to promote understanding (e.g., linear algebra courses can use technology for matrix manipulation or for visualizing the effects of linear transformations in two or three dimensions, and technology makes large linear systems tractable).
- for students in the mathematical sciences, progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof; gain experience in careful analysis of data; and become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.

The successful Linear Algebra student should be able to:

1. Understand algebraic and geometric representations of vectors in R*n* and their operations, including addition, scalar multiplication and dot product. understand how to determine the angle between vectors and the orthogonality of vectors.*
2. Solve systems of linear equations using Gauss-Jordan elimination to reduce to echelon form. Solve systems of linear equations using the inverse of the coefficient matrix when possible. Interpret existence and uniqueness of solutions geometrically.*
3. Perform common matrix operations such as addition, scalar multiplication, multiplication, and transposition. Discuss associativity and noncommutativity of matrix multiplication.*
4. Discuss spanning sets and linear independence for vectors in R*n*. For a subspace of R*n*, prove all bases have the same number of elements and define the dimension. Prove elementary theorems concerning rank of a matrix and the relationship between rank and nullity.*
5. Interpret a matrix as a linear transformation from R*n* to R*m*. Discuss the transformation’s kernel and image in terms of nullity and rank of the matrix. Understand the relationship between a linear transformation and its matrix representation, and

explore some geometric transformations in the plane. Interpret a matrix product as a composition of linear transformations.*

6. Use determinants and their interpretation as volumes. Describe how row operations affect the determinant. Analyze the determinant of a product algebraically and geometrically.*
7. Define eigenvalues and eigenvectors geometrically. Use characteristic polynomials to compute eigenvalues and eigenvectors. Use eigenspaces of matrices, when possible, to diagonalize a matrix.*
8. Use axioms for abstract vector spaces (over the real or complex fields) to discuss examples (and non-examples) of abstract vector spaces such as subspaces of the space of all polynomials.*
9. Discuss the existence of a basis of an abstract vector space. Describe coordinates of a vector relative to a given basis. For a linear transformation between vector spaces, discuss its matrix relative to given bases. Discuss how those matrices changes when the bases are changed.
10. Discuss orthogonal and orthonormal bases, Gram-Schmidt orthogonalization, orthogonal complements and projections. Discuss rigid motions and orthogonal matrices.
11. Discuss general inner product spaces and symmetric matrices, and associated norms. Explain how orthogonal projections relate to least square approximations.

9, 10 , 11 are optional in Ohio Transfer Module.