# MATH 14001 (10771) BASIC MATHEMATICAL CONCEPTS I (Plus)

## Upon successful completion of this course, the student will be able to:

#### Number

- 1a. Discuss the intricacies of learning to count, including the distinction between counting as a list of numbers in order and counting to determine a number of objects, and use pairings between elements of two sets to establish equality or inequalities of cardinalities.
- 1b. Attend closely to units (e.g., apples, cups, inches, etc.) while solving problems and explaining solutions.
- 1c. Discuss how the base-ten place value system (including extending to decimals) relies on repeated bundling in groups of ten and how to use objects, drawings, layered place value cards, base-ten blocks, and numerical expressions (including integer exponents) to help reveal base-ten structure.
- 1d. explain the development of fractions
- # Start with a whole.
- # Understand the fraction 1/b as one piece when the whole is divided into b equal pieces.
- # Understand the fraction a/b as a pieces of size 1/b and that the fraction a/b may be larger than one.
- # Understand fractions as numbers that can be represented in a variety of ways, such as with lengths (esp. number lines), areas (esp. rectangles), and sets (such as a collection of marbles).
- # Use the meaning of fractions to explain when two fractions are equivalent
- 1e. Model positive versus negative numbers on the number line and in real-world contexts.
- 1f. Reason about the comparison (=, <, >) of numbers across different representations (such as fractions, decimals, mixed numbers, ...).
- 1g. Demonstrate the skill of calculating simple arithmetic problems WITHOUT the use of a calculator.

#### **Operations**

- 2a. Recognize addition, subtraction, multiplication, and division as descriptions of certain types of reasoning and correctly use the language and notation of these operations.
- 2b. Illustrate how different problems are solved by addition, subtraction, multiplication and division and be able to explain how the operation used is connected to the solving of the problem.
- 2c. Recognize that addition, subtraction, multiplication, and division problem types and associated meanings for the operations (e.g., CCSS, pp. 88–89) extend from whole numbers to fractions and decimals.
- 2d. Employ teaching/learning paths for single-digit addition and associated subtraction and single-digit multiplication and associated division, including the use of properties of operations (i.e., the field axioms)
- 2e. Compare and contrast standard algorithms for operations on multi-digit whole numbers that rely on the use of place-value units (e.g., ones, tens, hundreds, etc.) with mental math methods students generate.
- 2f. Use math drawings and manipulative materials to reveal, discuss, and explain the rationale behind computation methods.
- 2g. Extend algorithms and mental math methods to decimal arithmetic.
- 2h. Use different representations of the same fraction (e.g., area models, tape diagrams) to explain procedures for adding, subtracting, multiplying, and dividing fractions. (This includes connections to grades 6–8 mathematics.).
- 2i. Explain the connection between fractions and division,  $a/b = a \div b$ , and how fractions, ratios, and rates are connected via unit rates. (This includes connections to grades 6–8 mathematics. See the Ratios and Proportional Relationships Progression for a discussion of unit rate.).
- 2j. Explain why the extensions of the operations to signed numbers make sense.

#### **Algebraic Thinking**

- 3a. Model and communicate their reasoning about quantities and the relationships between quantities using a variety of representations.
- 3b. Discuss the foundations of algebra in elementary mathematics, including understanding the equal sign as meaning "is the same [amount] as" rather than a "calculate the answer" symbol.
- 3c. Look for regularity in repeated reasoning, describe the regularity in words, and represent it using diagrams and symbols and communicate the connections among these.
- 3d. Articulate, justify, identify, and use properties of operations.
- 3e. Describe numerical and algebraic expressions in words, parsing them into their component parts, and interpreting the components in terms of a context.
- 3f. Use a variety of methods (such as guess and check, pan balances, strip diagrams, and properties of operations) to solve equations that arise in "real-world" contexts.

### **Number Theory**

- 4a. Demonstrate knowledge of prime and composite numbers, divisibility rules, least common multiple, greatest common factor, and the uniqueness (up to order) of prime factorization
- 4b. Discuss decimal representation and recognize that there are numbers beyond integers and rational numbers.