

# Design and Analysis of Algorithms

## **Problem #1.**

Let  $A$  and  $B$  be two sequences of  $n$  integers each. Given an integer  $m$ , describe an  $O(n \log n)$  time algorithm for determining if there is an integer  $a$  in  $A$  and an integer  $b$  in  $B$  such that  $m = 2a + 3b$ .

## **Problem #2.**

Demonstrate the insertion of the keys  $5, 28, 19, 15, 20, 33, 12, 17$  into a hash table with collisions resolved by *linear probing*. Assume that the table has 9 slots and that the hash function is  $h(k) = k \bmod 9$ . Draw the state of the hash table after every insertion. Propose strategies other than *linear probing* for handling *collisions* in a hash table. Are they better, worse? Explain.

## **Problem #3.**

Suppose you are given a diagram of a telephone network, which is a graph  $G$  whose vertices represent switching centers, and whose edges represent communications lines between two centers. The edges are marked by their bandwidth. The bandwidth of a path is the bandwidth of its lowest bandwidth edge. Describe an algorithm that, given a diagram and two switching centers  $a$  and  $b$ , will output the maximum bandwidth of a path between  $a$  and  $b$ . (Just report the maximum bandwidth; you do not have to give the actual path). Analyze the running time of your algorithm.

## **Problem #4.**

Each of  $n$  keys in an array may have one of the values *red*, *white* or *blue*. Give a linear time algorithm for rearranging the keys so that all the reds come before all the blues and all the blues come before all the whites. The only operations permitted on the keys are

- Examination of a key to find out what color it is.
  - A swap (interchange of positions) of two keys specified by their indices.
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