Ph.D. Preliminary Examinations
Design and Analysis of Algorithms
January 2007

Design and Analysis of Algorithms (Questions 5 – 8)

Problem 5

[SORTING] You need to find the **k-th** key inside an unsorted list. The problem is simpler than sorting the full list. Give a simplification of Quick Sort algorithm that finds the **k-th** key with less cost.

Your presentation of this algorithm should:

(i) outline the solution idea

(ii) give a pseudocode level statement of the algorithm (using the critical variables)

(iii) analyze the average cost.

(10+60+30 points)

Problem 6

[ASYMPTOTICS] (i) Define Big Omega and Big Oh bounds of a function. (ii) A function f(n) belongs to the set $\Theta(g(n))$ if there exist positive constants c1 and c2 such that it can be bonded from top by c1.g(n) and bounded from below by c2.g(n), for sufficiently large n. Prove whether $2n^2 - 3n = \Theta(n^2)$. If it is false, then show that no such c1, c2, and n can exist. If it is true, then show at least one such set of C1, C2, and n.

(40+60 points)

Problem 7

[Dynamic Programming]: In a shopping contest, the shopper who can gather items of highest total value in their shopping cart is allowed to take them all free. Each shopper starts with a shopping cart with a maximum capacity W pounds. The store has n items of value v_i and weight w_i. (100 + 50+50 =200 points) (i) Develop dynamic programming style O(nW) algorithm to win this contest. (ii) What are the two critical problem characteristics that make a problem suitable for dynamic programming? Explain your answer by identifying them in the above solution.

(60+40 points)

Problem 8

[TREE] Define an AVL tree. Derive an upper bound on the height of AVL tree. (The tighter the bound you can derive the better you will score)

(30+70 points).