August 2008
Preliminary Exam

Design & Analysis of Algorithms (Questions 5 – 8)

Problem 5

NASA wants to link n stations spread over the country using communication channels. Each pair of stations has a different bandwidth available, which is known a priori. NASA wants to select $n-1$ channels (the minimum possible) in such a way that all the stations are linked by the channels and the total bandwidth (defined as the sum of the individual bandwidths of the channels) is maximum. Give an efficient algorithm for this problem and determine its worst-case time complexity.

Problem 6

Let A and B be two sequences of n integers each. Given an integer m, describe an $O(n \log n)$ time algorithm for determining if there is an integer a in A and an integer b in B such that $m = a + b$.

Problem 7

Let T be a binary search tree, and let x be a key. Give an efficient algorithm for finding the smallest key y in T such that $y > x$. Note that x may or may not be in T. Explain why your algorithm has the running time it does.

Problem 8

Suppose you are given a diagram of a telephone network, which is a graph G whose vertices represent switching centers, and whose edges represent communications lines between two centers. The edges are marked by their bandwidth. The bandwidth of a path is the bandwidth of its lowest bandwidth edge. Describe an algorithm that, given a diagram and two switching centers a and b, will output the maximum bandwidth of a path between a and b. (Just report the maximum bandwidth; you do not have to give the actual path). Analyze the running time of your algorithm.