August 2007
Preliminary Exam

Design & Analysis of Algorithms (Questions 5 – 8)

<u>Problem 5</u>

(i) Explain what is a B-tree. (ii) Explain the insertion and deletion procedures for a node in a B tree and (iii) prove that these do not violate the B-tree properties.

<u>Problem 6</u>

Provide worst-case analysis of the number of comparisons required in (a) Quick Sort and (b) in Merge Sort, and show which one is better.

<u>Problem 7</u>

a) Prove that in a hash table in which collisions are resolved by chaining, a successful search takes time $O(1+L)$, where L is the load factor, under the assumption of simple uniform hashing (Hint: note that the analysis for successful search is different from that of unsuccessful search, and you may not get any points for showing it for the latter).

b) What is clustering? Can you explain with probabilistic analysis why it occurs for linear probing? If yes, give your explanation, if no explain why it cannot be done.

<u>Problem 8</u>

There are n trading posts along a river situated on both banks. At any of the posts you can rent a canoe to be returned at any other posts downstream (paddling upstream is impossible). For each possible departure point i and each possible arrival point j, the cost of a rental from i to j is given. However, it can happen that the cost of renting from i to j is higher than the total cost of a series of shorter rentals between the same posts. In this case you however can simply return the first canoe at some post k between i and j and continue your journey in a second canoe.

   (i)   Give an efficient algorithm to determine the costs of a trip by canoe from each possible departure point i to each possible arrival point j.
   (ii)   Derive the complexity of the algorithm in terms of n.
   (iii)   What are the two critical problem characteristics that make a problem suitable for dynamic programming? Explain if you can identify them in the above problem?